Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Control for Energy Dissipation in Structures

S. P. Joshi,* T. L. Vincent,† and Y. C. Lin‡
University of Arizona, Tucson, Arizona 85721

Introduction

ACTIVE vibration suppression of a large flexible space structure has been studied extensively in recent years. The studies include reduced-order modeling of structures with associated controller design. The main objectives of active control design are minimizing hardware and real-time computation while achieving efficient and robust control of the structure. Structure modeling and various control techniques are briefly discussed in this paper. Control designs based on state variable feedback are compared with an energy dissipation-based design. The effect of nonlinearities is considered by allowing saturation of the actuator control. The energy dissipation control design is shown to be superior to others in light of the aforementioned objectives.

There have been a number of control schemes proposed for flexible structure control. Common methods of control are state space methods, which include pole allocation[1] and the linear quadratic performance methods[2]; direct velocity feedback[3] control, which is part of the collocated control[4]; and nonlinear control.[5] Another method of control known as independent modal space control is extensively researched in recent literature.[6] This method is based on modal space representation and can be considered as a special case of the state space methods.

The focus of traditional state variable feedback methods has been on system stability (eigenvalue placement). Since the origin is also at a zero energy state, a stable system will achieve the main objective of vibration suppression. The direct velocity feedback control, which will hereafter be referred to as energy dissipation control because it is based on system energy, guarantees stability properties in the presence of structural uncertainties.[4] The stability is guaranteed regardless of the reduced-order model and uncertainties in the knowledge of the parameters. It has been shown that the closed-loop system is always stable in the sense of Lyapunov and is also asymptotically stable under certain additional conditions.[3,4] The controller is also shown to be robust with respect to nonlinearities, phase shifts, and delays.[7]

This paper presents a direct comparison of the state variable feedback control and energy dissipation control by numerical simulation. The effect of nonlinearities is considered by allowing saturation of the actuator control force.

Presented as Paper 88-2272 at the AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18-20, 1988; received July 25, 1988; revision received April 12, 1989.

*Assistant Professor, Department of Aerospace and Mechanical Engineering; currently at Aerospace Engineering Department, University of Texas at Arlington. Member AIAA.

†Professor, Department of Aerospace and Mechanical Engineering. Member AIAA.

‡Graduate Student, Department of Aerospace and Mechanical Engineering.

Structural System Formulation

A flexible structure is modeled either as a continuous system or as a discrete system, depending on the nature of the problem. The discrete system formulation has many commonly used representations; among them is the finite-element method. Application of the finite-element method to a conservative dynamical system results in a system of second-order differential equations[8]

$$M\ddot{Y} + KY = F \tag{1}$$

where M and K are mass and stiffness matrices, respectively, F is a generalized nodal force vector, and Y is a generalized displacement vector. For a beam,

$$Y = [y_1, \theta_1, \ldots, y_n, \theta_n]^T \tag{2}$$

where y_i and θ_i are transverse displacement and rotation associated with the ith node. A system of first-order differential equations, in a state space representation, may now be obtained. We first define a momentum vector P.

$$P = M\dot{Y} \tag{3}$$

We then relate the generalized force vector F to the vector of physical actuator input

$$U = [u_1, q_1, \ldots, u_n, q_n]^T$$

where u_i and q_i are the force and torque, respectively, applied at node i. This is done by means of an actuator placement matrix, so that

$$F = BU$$

The matrix B specifies where the force and torque actuators are located. It is a diagonal matrix, with the diagonal composed of zeros (no actuators) and ones (actuators). Substituting in Eq. (1), we obtain

$$\begin{Bmatrix} \dot{Y} \\ \dot{P} \end{Bmatrix} = \begin{bmatrix} O & M^{-1} \\ -K & O \end{bmatrix} \begin{Bmatrix} Y \\ P \end{Bmatrix} + \begin{Bmatrix} O \\ B \end{Bmatrix} U \tag{4}$$

Control Design

Two controller designs are described in this section. The first one is a traditional optimal state feedback control design. The second one is a design based on energy dissipation. The second design is also known as direct velocity feedback control.

Optimal State Feedback Control

Optimal state feedback control design for a linear system dynamics, of the form $\dot{X} = AX + BU$, is based on the idea of choosing a control law that will minimize the quadratic performance index, of the form

$$J = \int_0^\infty \left[X^T(t)QX(t) + U^T(t)RU(t) \right] \mathrm{d}t \tag{5}$$

where X is the state vector, U is the control vector, and Q and R are constant matrices. It is assumed that Q is positive semidefinite and R is positive definite. If $U(t)$ is not constrained, then an optimal feedback control exists, is unique and is of the form

$$u(t) = -R^{-1}\tilde{B}^T\hat{K}X(t) \tag{6}$$

where the constant, symmetric, and positive definite matrix $\hat{K}$ is the solution of the nonlinear time-invariant matrix algebraic equation

$$\hat{K}A + \tilde{A}^T\hat{K} - \hat{K}\tilde{B}R^{-1}\tilde{B}^T\hat{K} = -Q \tag{7}$$

Energy Dissipation Control

For a conservative system, like an ideal beam structure, the kinetic and potential energy can be written as

$$T = \tfrac{1}{2}\dot{Y}^T M \dot{Y} \tag{8}$$

$$V = \tfrac{1}{2}Y^T K Y \tag{9}$$

where T is the kinetic energy, V the potential energy, Y the generalized displacement vector (e.g., transverse displacement, rotation), and $\dot{Y}$ the generalized velocity vector. The total energy H is given by

$$H = T + V = \tfrac{1}{2}\dot{Y}^T M \dot{Y} + \tfrac{1}{2}Y^T K Y \tag{10}$$

To dissipate energy, we can choose

$$F = -S\dot{Y} \tag{11}$$

where S is a positive semidefinite (definite) matrix. It can easily be verified that

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -\dot{Y}^T S \dot{Y} \tag{12}$$

It follows from Eq. (12) that the controller given by Eq. (11) will never increase system energy. Except for some special cases (discussed in the next section), the total energy will always be reduced under this control action.

Control Design Comparison

The model considered for the comparison is a four-element finite-element model of an aluminum cantilever beam of dimensions $45 \times 1.5 \times 0.25$ in. The mass and stiffness distributions are $M(x) = 6.7633 \times 10^{-4}$ lb·s^2/in.^{2}, $EI(x) = 1.9336 \times 10^4$ lb in.^{2}. A velocity sensor and a control force actuator is assumed to be located 31 in. from the fixed end of the cantilever beam. We now compare an energy dissipation controller with an optimal state feedback controller under both unbounded and bounded control. An initial impulse of 0.02-s duration and 4 lbf is applied 31 in. from the fixed end in each case. Control action to damp out this disturbance is then initiated after 1 s. Two cases are simulated numerically.

1) Energy dissipation control with $u = 0.125\dot{y}_3$: $|u|$ unbounded; $|u| \le 0.5$.

2) Optimal state feedback control with $[Q] = [I]$; $[R] = [I]$: $|u|$ unbounded; $|u| \le 0.5$.

The cantilever beam displacement at the sensor location is plotted in Fig. 1 for the two cases with unbounded control. It is evident that the energy dissipation control brings the system back to equilibrium more quickly. The change in total energy after application of control forces (unbounded control) is

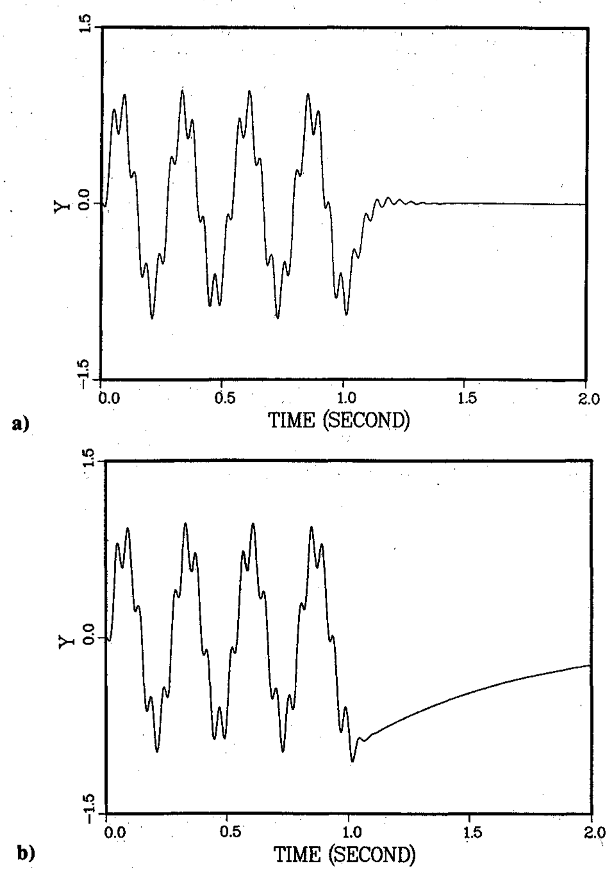

Fig. 1 Transverse displacement at the sensor location (unbounded control force): a) energy dissipation; b) state variable feedback.

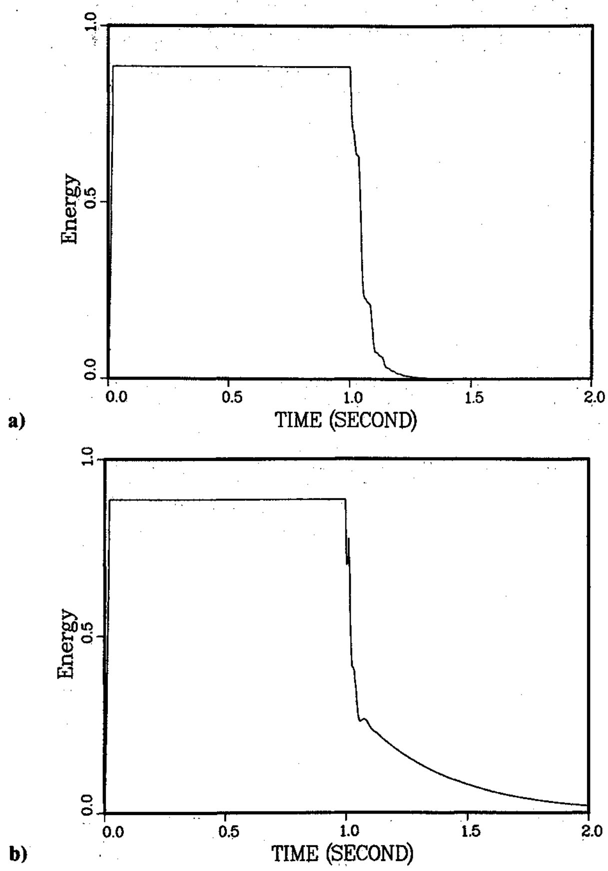

Fig. 2 Variation of the total energy in the beam (unbounded control force): a) energy dissipation; b) state variable feedback.

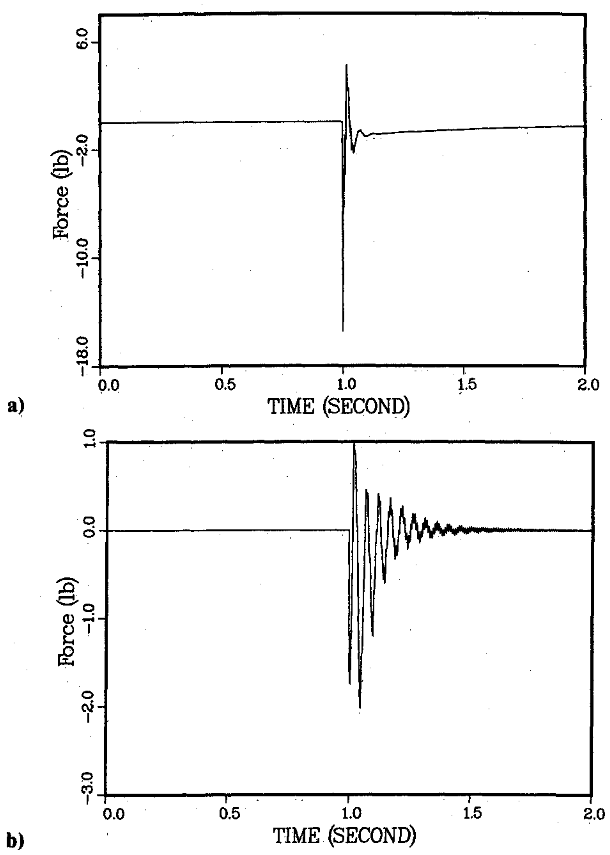

Fig. 3 Control force history (unbounded control force): a) energy dissipation; b) state variable feedback.

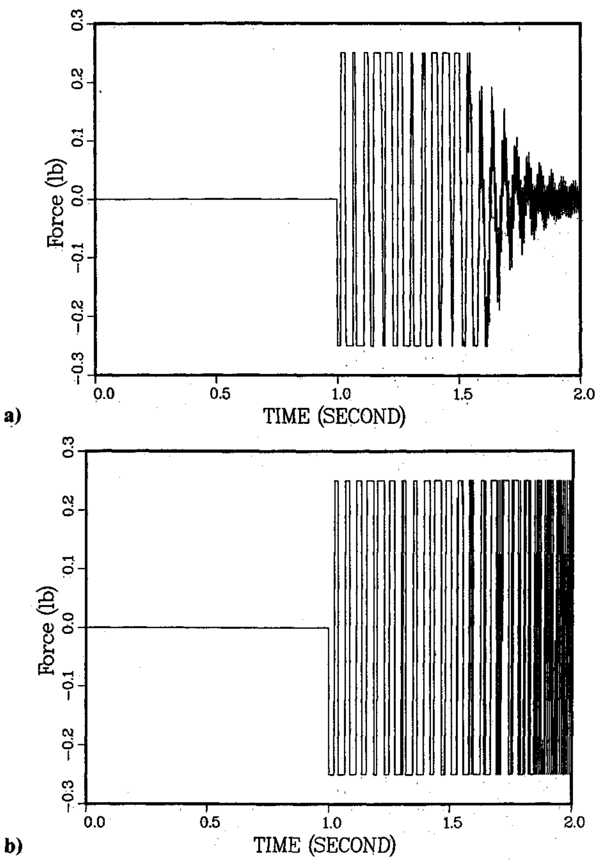

Fig. 4 Control force history (bounded control force, $|u| \leq 0.5$): a) energy dissipation; b) state variable feedback.

shown in Fig. 2. The total energy is more effectively reduced using energy dissipation. The control force history with unbounded control for the two cases is shown in Fig. 3. This may be compared with the bounded control case of Fig. 4, where state variable feedback produces a longer chattering control. More simulation results, along with some experimental observations, are provided in Ref. 9.

Although the energy dissipation controller may be only marginally better than the optimal state feedback-based controllers, its main advantage is that it can be implemented with only one sensor. State variable feedback requires (for the beam example discussed here) 16 sensors or a combination of sensors and an observer. The hardware requirements and/or on-line computation (required for an observer) for the state variable feedback control is considerable compared to the energy dissipation controller. In addition, the optimal state feedback control design is based on reduced-order modeling and is therefore sensitive to the accuracy of the model. In contrast, the energy dissipation control will never impart energy to the structure and is insensitive to the global behavior of the structure.

Conclusions

A comparison of optimal control and energy dissipation control discussed in this paper shows that the latter provides good control with simplicity. The results indicate that energy dissipation control achieves the same or better results than optimal control without the large number of sensors or an observer. The single fact that the energy dissipation controller reduces the hardware requirement drastically favors this control design. The structural system energy monotonically reduces in the case of the energy dissipation controller, whereas it may increase during control operation with other designs. A rapid increase in the total energy during control operation is not desirable in general because it may cause damage to a fragile structure.

The energy dissipation control design is insensitive to uncertainties in the physical structure. This is a desirable feature because any additional appendage to the original structure will not make the controller unstable.

Constraints on actuator forces (bounded control) are present in any real situation. The energy dissipation control design still performs adequately with the introduction of such nonlinearities.

References

[1]Dorf, R. C., *Modern Control Systems*, 3rd Ed., Addison-Wesley, Reading, MA, 1980.

[2]Athans, M., and Falb, P. L., *Optimal Control—An Introduction to Theory and Its Applications*, McGraw-Hill, New York, 1966.

[3]Balas, M. J., "Direct Velocity Feedback Control of Large Space Structure," *Journal of Guidance and Control*, Vol. 2, No. 3, 1979, pp. 252–253.

[4]Joshi, S. M., "A Controller Design Approach for Large Flexible Space Structures," NASA Cr-165717, May 1981.

[5]Meirovitch, L., Baruh, H., and Oz, H., "A Comparison of Control Techniques for Large Flexible Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 6, No. 4, 1983, pp. 302–310.

[6]Lindberg, R. E., Jr., and Longman, R. W., "On the Number and Placement of Actuators for Independent Modal Space Control," *Journal of Guidance, Control, and Dynamics*, Vol. 7, No. 2, 1984, pp. 215–221.

[7]Joshi, S. M., "Robustness Properties of Collocated Controllers for Flexible Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 9, No. 1, 1986, pp. 85–91.

[8]Meirovitch, L., *Elements of Vibration Analysis*, McGraw-Hill, New York, 1975.

[9]Joshi, S. P., Vincent, T. L., and Lin, Y. C., "Control for Energy Dissipation in Structures," AIAA Paper 88-2272, April 1988.